Generalized Geertsma solution for ground deformation

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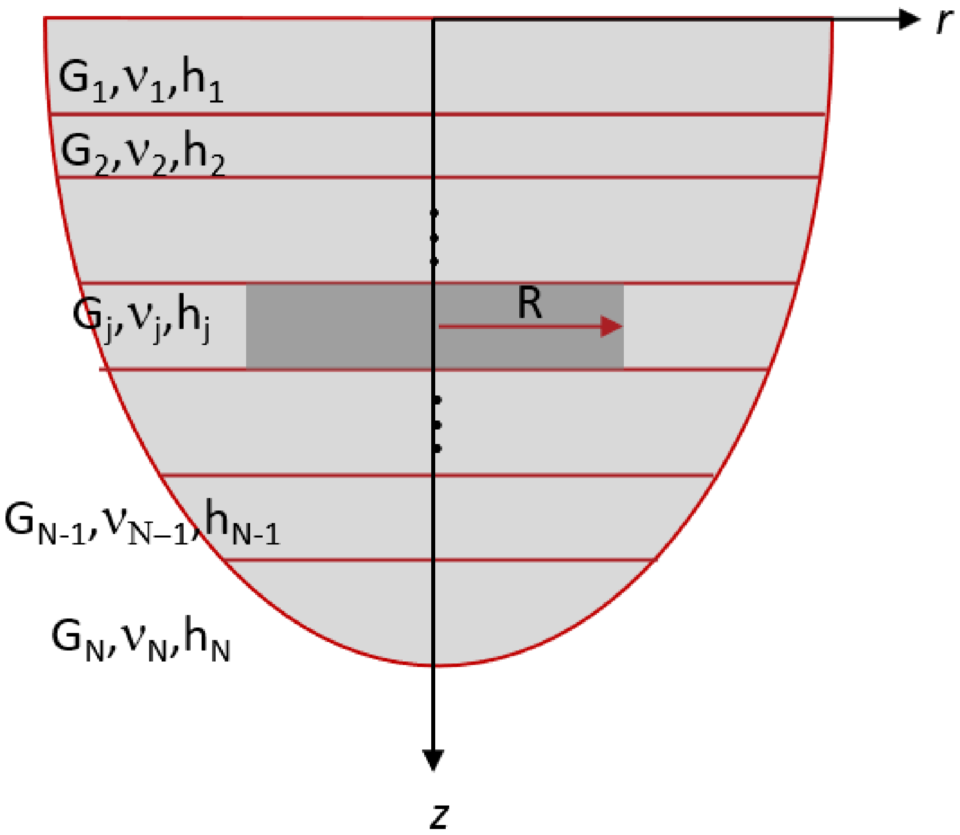
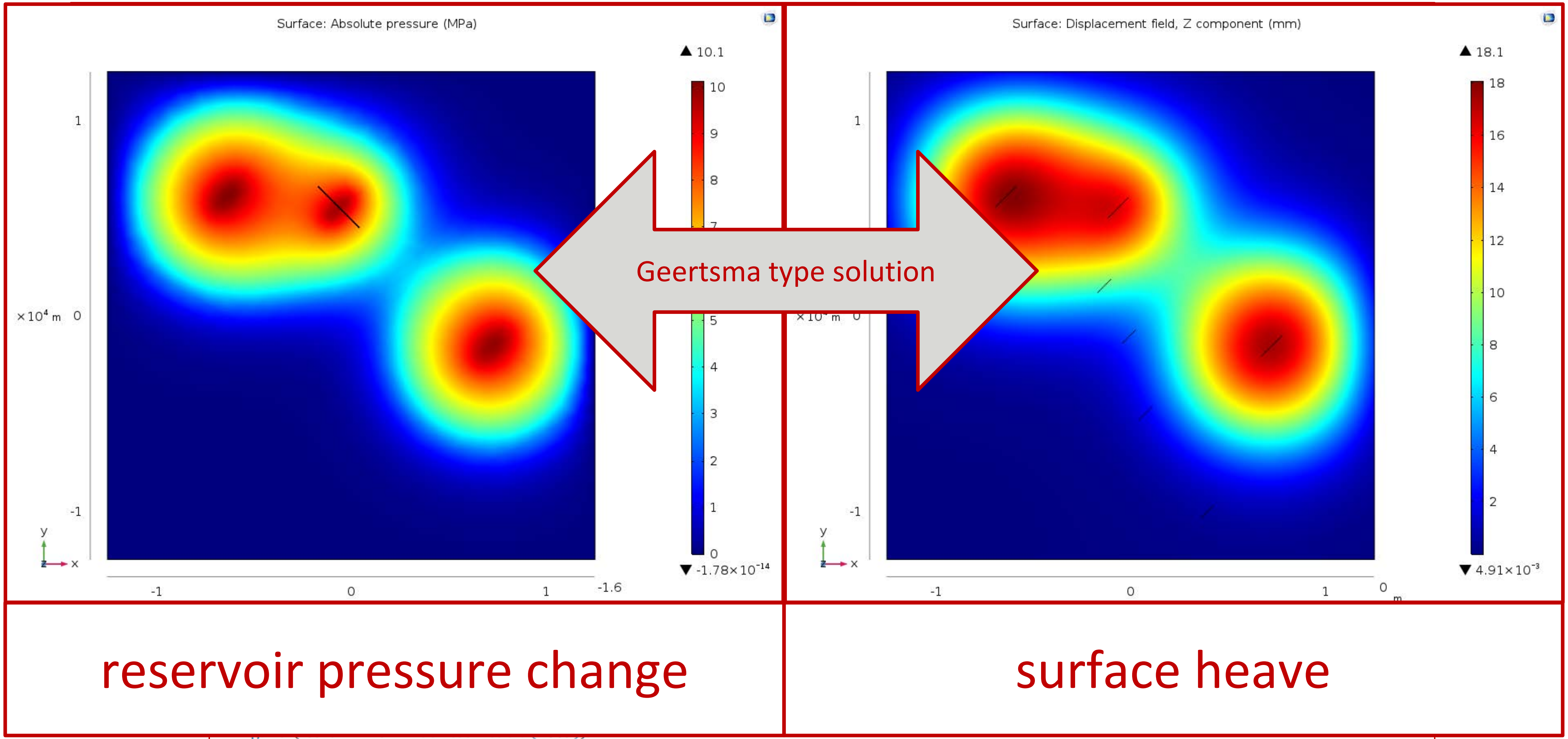
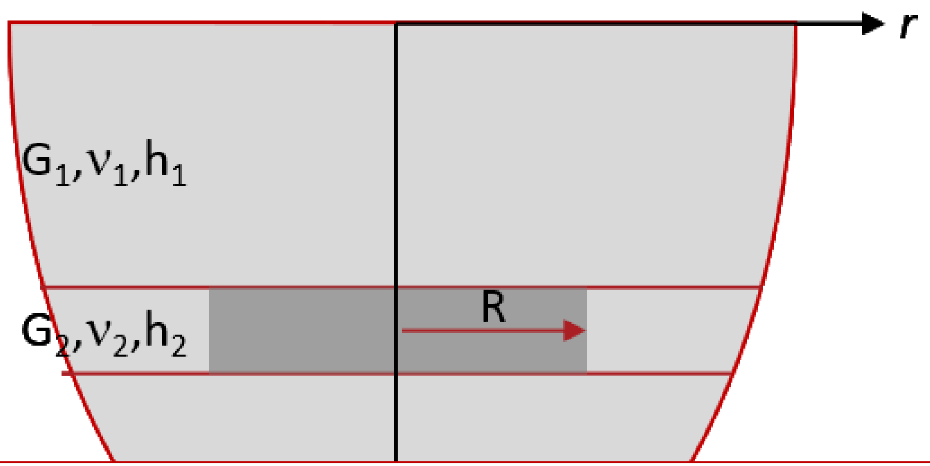


Background and Motivation (In Salah experience)



Brief history on Geertsma type solutions



$$u_z(r,0) = -2c_m(1-\nu)\,\Delta p\, H\, R \int_0^\infty e^{-D\alpha} J_1(\alpha R)\, J_0(\alpha r)\, d\alpha\,, \qquad (6)$$

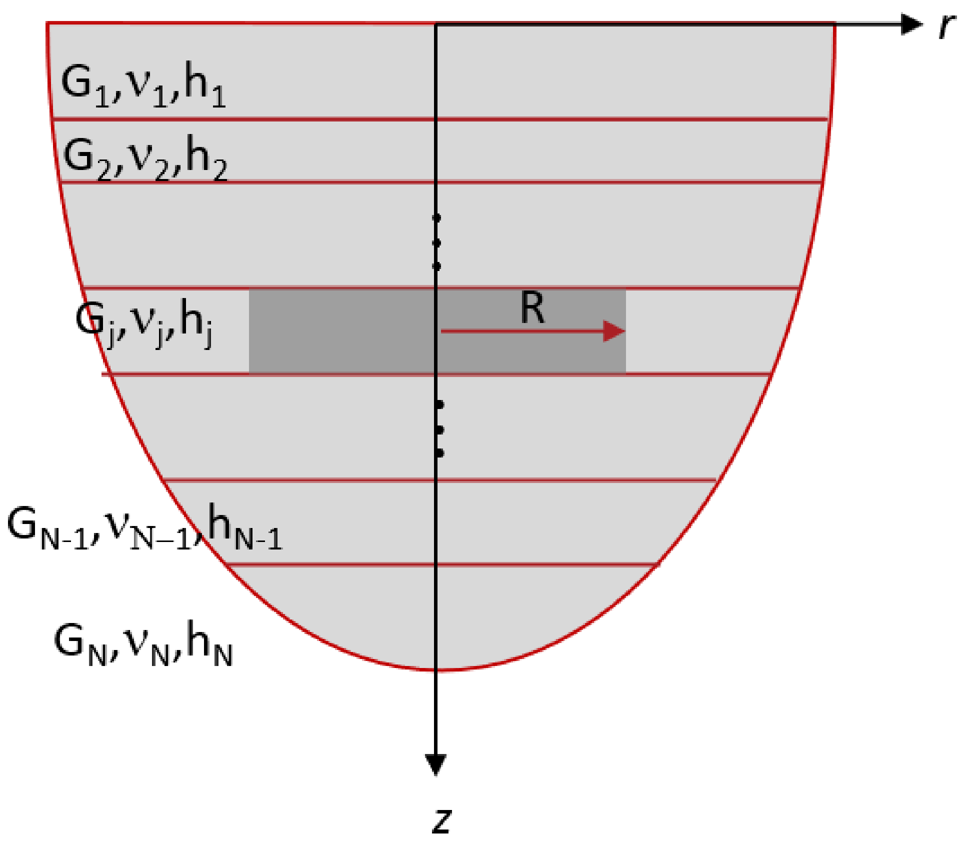
and

$$u_r(r,0) = +2c_m(1-\nu)\,\Delta p\, H\, R \int_0^\infty e^{-D\alpha} J_1(\alpha R)\, J_1(\alpha r)\, d\alpha\,, \qquad (7)$$

- Geertsma +++ (1970’s and earlier)
- Tempone et al. (2010)
- Mehrabian and Abousleiman (2015)
- Wangen and Halvorsen (2019)
- Etc.

- All those above require numerical integration of Hankel transform.

Generalized Geertsma solution from SENSE



- Any number and thickness of layers can be simulated.
- We can calculate deformation and stress at any layer for «static» pressure or temperature distribution applied at any layer.
- Any boundary condition is available e.g. rigid basement (e.g. Tempone et al., 2010).
- Matlab and Python scripts are implemented.
- Anisotropy medium model can also be considered i.e. $G_h/G_v \neq 1$. (Park et al. 2021)

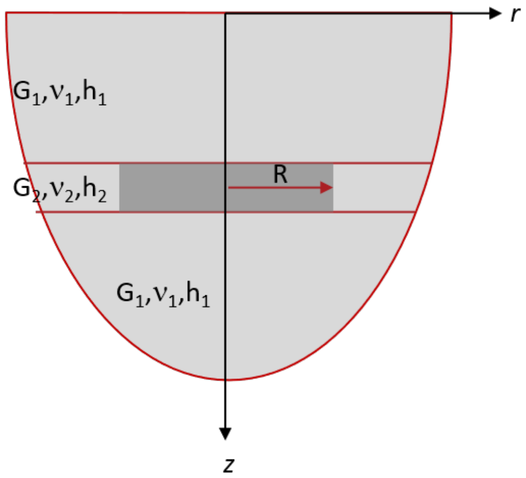
Derivation (some math needed ☺)

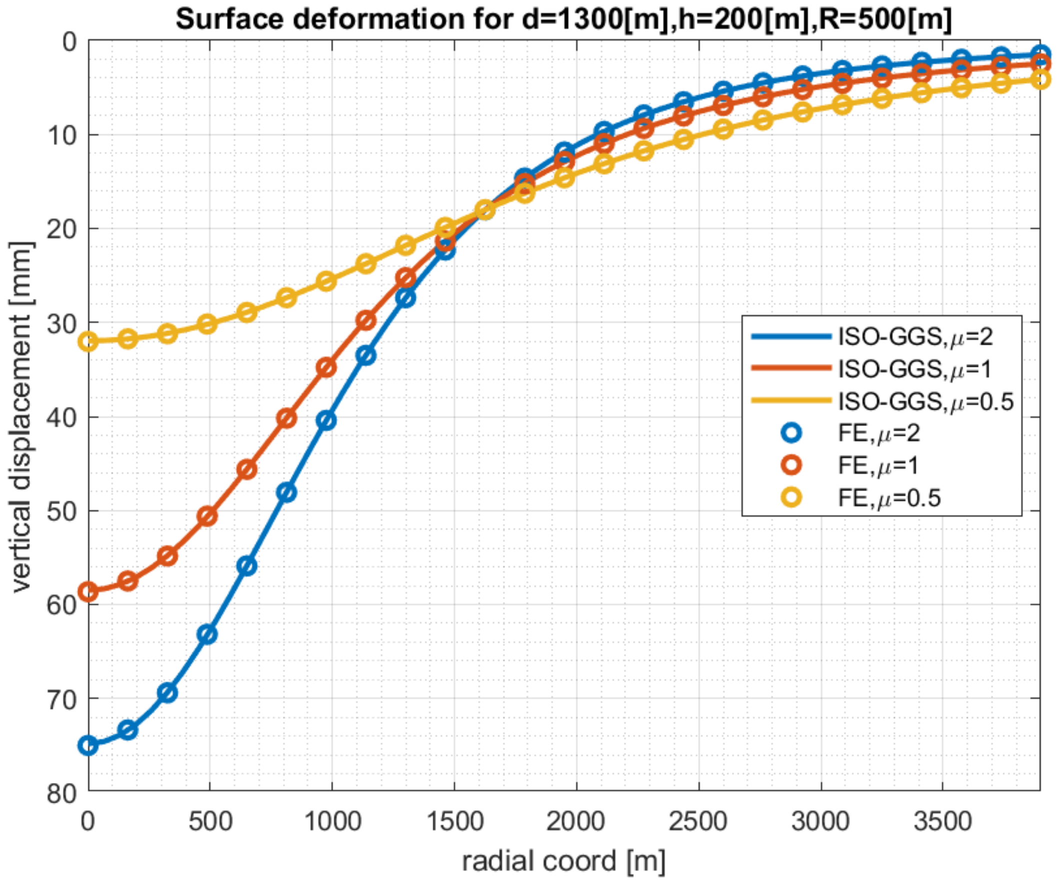
- $U_1 = \frac{c_m P}{k} + az\left(Ae^{kz} - Be^{-kz}\right) + Ce^{kz} + De^{-kz}$
- $U_3 = \left(\frac{a+1}{k} - az\right) Ae^{kz} - \left(\frac{a+1}{k} + az\right) Be^{-kz} - Ce^{kz} + De^{-kz}$
- $S_{rz} = 2G\left[\left(akz - \frac{1}{2}\right) Ae^{kz} + \left(akz + \frac{1}{2}\right) Be^{-kz} + kCe^{kz} - kDe^{-kz}\right]$
- $S_{zz} = 2G\left[\left(1 - akz + \frac{\nu}{1-2\nu}\right) Ae^{kz} + \left(1 + akz + \frac{\nu}{1-2\nu}\right) Be^{-kz} - kCe^{kz} - kDe^{-kz} - c_m P\right]$
- where $a = 1/2(1-2\nu)$; $c_m = \alpha(1-2\nu)/2G\nu$; $P = R/kJ_1(kR)$
- A,B,C,D for each layer should be determined to satisfy the BC and IC along the layered medium.

Validation wrt FE solution

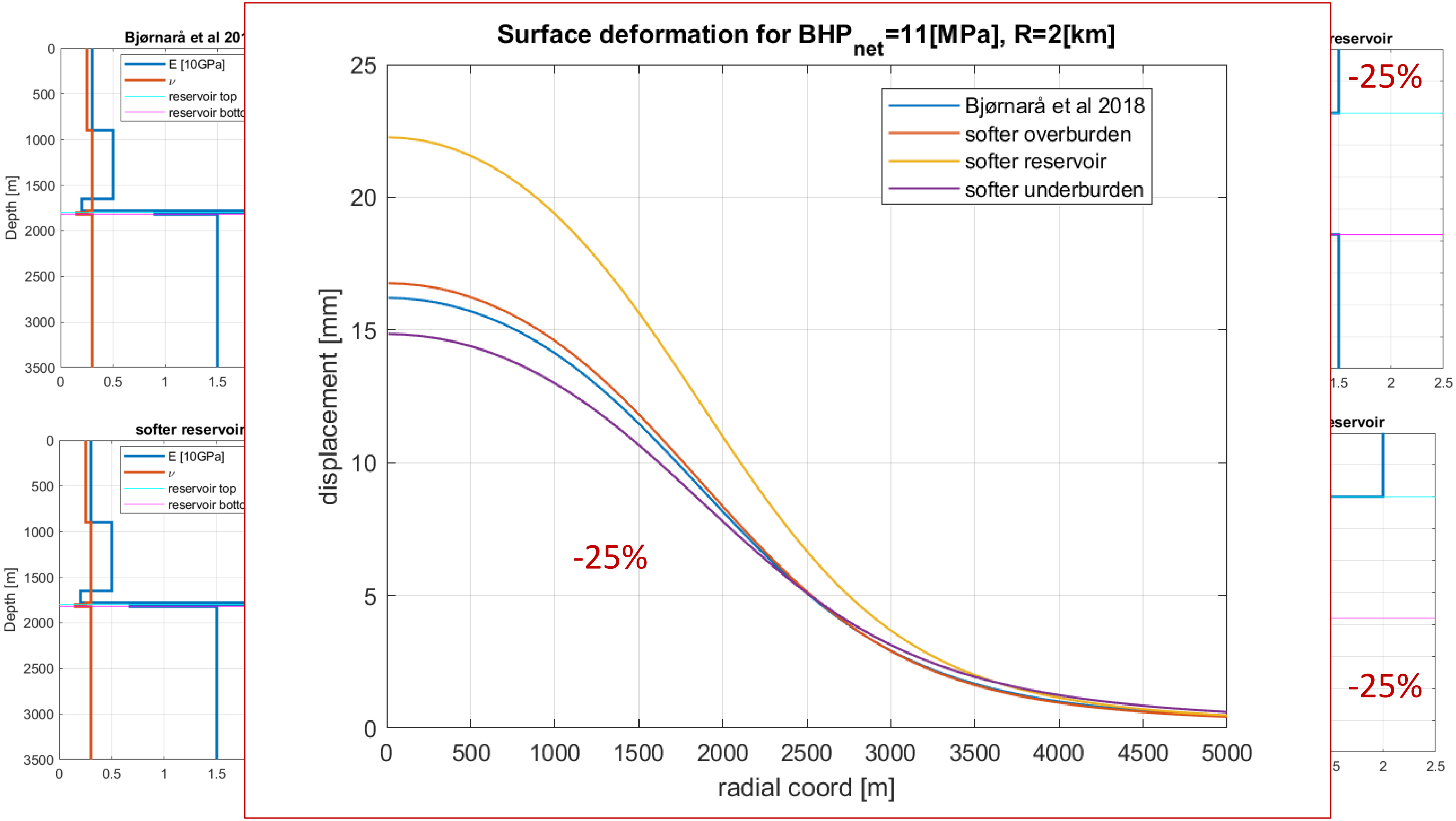
layer	thickness [m]	shear modulus (*G*) [GPa]		
		model 1	model 2	model 3
1	1300	0.5	1.0	2.0
2	200	1.0	1.0	1.0
3	∞	0.5	1.0	2.0

Poisson's ratio is 0.25 for all the layers; $\mu=G_1/G_2$

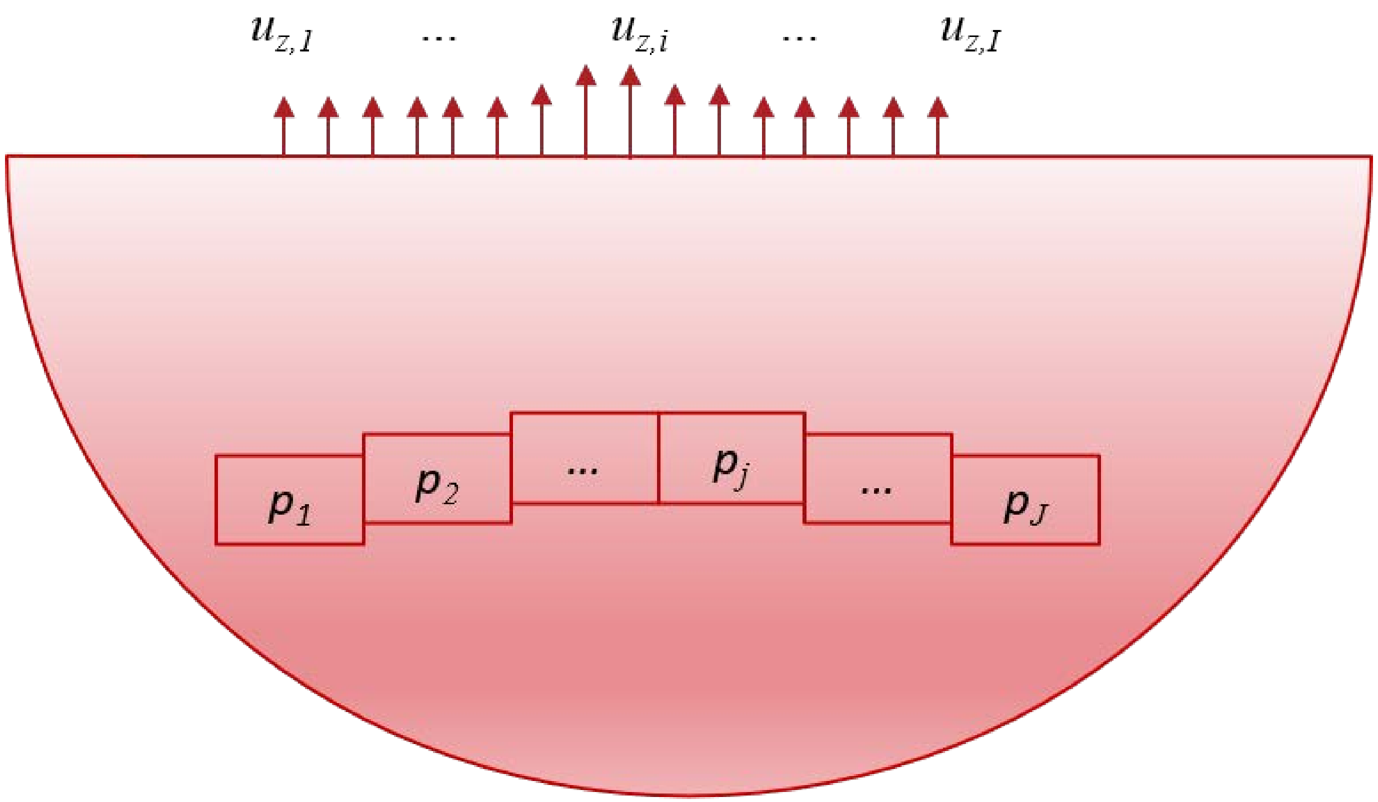




Effects of layering

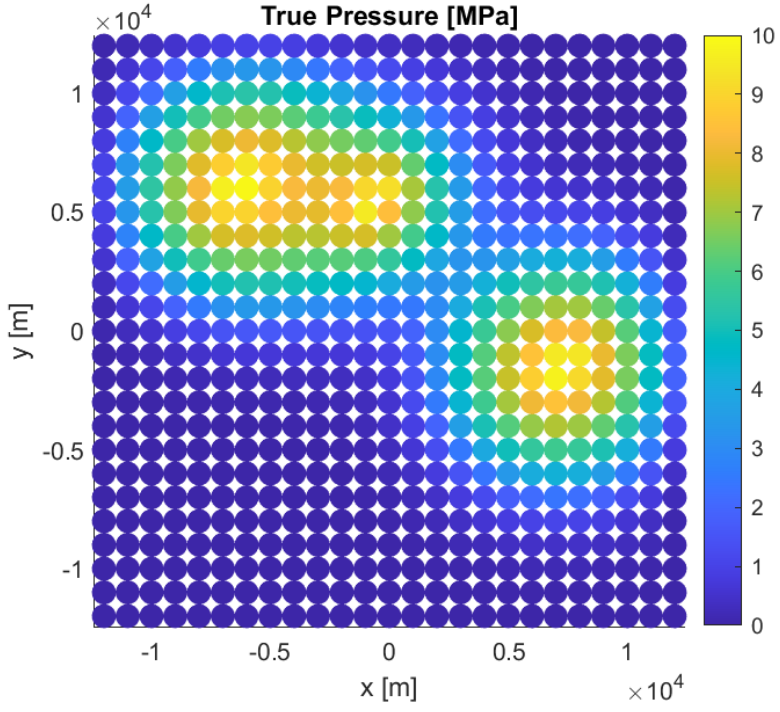


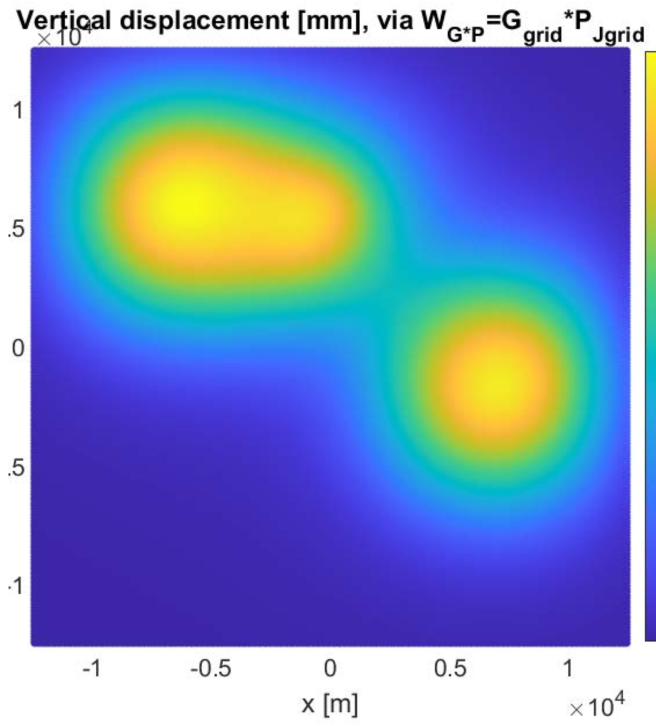
For realistic pressure distribution

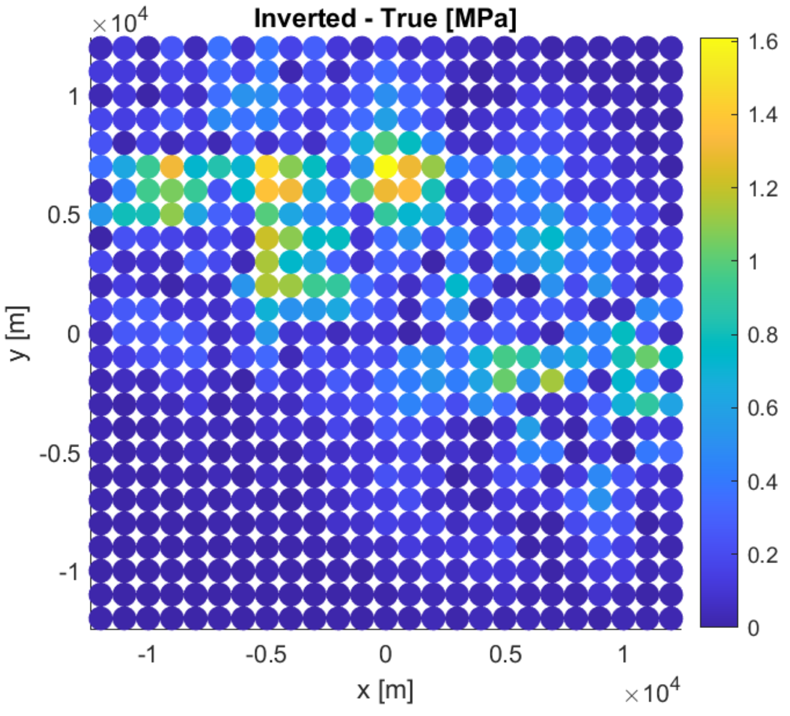


- $u_{z,i} = \sum_{j=1}^{J} g_{z,ij} p_j$
- $\mathbf{U}_z = \mathbf{G}_z \mathbf{P}$
  - $\mathbf{U}_z = [u_{z,1}, u_{z,2}, \ldots\ u_{z,i}, \ldots, u_{z,I}]^T$,
  - $\mathbf{P} = [p_1, p_2, \ldots, p_j, \ldots, p_J]^T$,
  - $\mathbf{G}_z = \begin{bmatrix} g_{z,11} & \cdots & g_{z,1J} \\ \vdots & \ddots & \vdots \\ g_{z,I1} & \cdots & g_{z,IJ} \end{bmatrix}$
- $\mathbf{P} = \mathbf{G}_z^{-1} \mathbf{U}_z$

Effects of heave data noise via synthetic data







$$\mathbf{G}_z\ \mathbf{P} = \mathbf{U}_z$$

$$\mathbf{G}_z^{-1}\mathbf{U}_{z,3\%} = \mathbf{P}$$

Summary and conclusion

- A generalized Geertsma solution is presented to handle arbitrary number, depth and thickness of isotropic homogeneous layers.
  - The solution is validated by comparing the analytical solution to a numerical solution.
- We have applied the generalized solution to various subsurface models to study the effect of subsurface layering on surface deformation and the impact of acquisition-processing accuracy on inverted reservoir pressure.
  - It is shown that, for the tested case-study inspired by the In Salah CO2 storage project, the surface deformation is dependent on the mechanical properties, particularly of reservoir.
  - Finally, the inversion exercise has demonstrated that 3% noise due to acquisition-processing error may introduce up to 15% error in the inverted pressure
- This fast calculation engine is applied for more advanced inversion approach e.g. via ML + 2-step inversion (done by Jean-Remi Dujardin and Hector Marin Moreno at NGI).

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References

TCCS-11 - Trondheim Conference on CO_2 Capture, Transport and Storage
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Trondheim CCS Conference
CO_2 Capture, Transport and Storage

GENERALIZED GEERTSMA SOLUTION FOR ISOTROPIC LAYERED MEDIUM

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Article

An Analytical Solution for Pressure-Induced Deformation of Anisotropic Multilayered Subsurface

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